

Quasar lensing

Introduction

Strong gravitational lensing is a phenomenon which occurs when the lines of sight to a foreground and background object nearly coincide, resulting in multiple imaging of the background object. Although quite rare, it offers an important diagnostic of masses and mass distributions in foreground objects ranging from stars to clusters of galaxies, and has the important advantage of being sensitive to all kinds of matter, both baryonic and dark. Strong lensing also gives magnified views of background objects, allowing easy study of otherwise inaccessible quantities, and potentially cosmological information owing to its sensitivity to combinations of mass density and lengths within the universe.

Quasars are relatively rare phenomena, and lensed quasars, in which a foreground galaxy provides the gravitational deflection in the right place, are correspondingly rare. However, they have some unique advantages: quasars allow easy access to the high redshift universe; quasars are bright, allowing easy study of subtle effects and potentially allowing complete samples to be built; quasars are variable, allowing cosmological information to be derived from time delays; quasars emit at multiple wavelengths, allowing detailed study of propagation effects. This review gives an overview of the history of quasar lensing, and a summary of its main applications. The applications fall into three parts: the use of quasars as probes of lens galaxies, both their stellar content via microlensing and their dark matter content via the fitting of models to lensed images; the use of quasar lenses to probe the structure and properties of the quasars themselves; and the use of quasar lenses for cosmology. A number of previous reviews have addressed some or all of these issues; see, for example Wambsganss (1998), Claeskens & Surdej (2002), Courbin, Saha & Schechter (2002), Kochanek & Schechter (2004), Kochanek (2004), Wambsganss (2004), Jackson (2007), Zackrisson & Riehm (2010), Bartelmann (2010), Schmidt & Wambsganss (2010). In this review individual theoretical results will be presented as needed, without derivation; the interested reader can refer to the standard text by Schneider, Ehlers & Falco (1992) for more detail. Finally I outline the possible future applications of quasar lensing, and the observational programmes which will develop the subject in the coming years.

Historical introduction

Lensing remained an interesting theoretical possibility for most of the twentieth century. The deflection of light by a mass was calculated classically by Soldner (1801) and correctly by Einstein as a consequence of general relativity, and famously observed by Eddington in 1919 using position shifts of stars close to the Sun during a solar eclipse. The possibility of multiple images formed by individual objects was considered by Chwolson (1924). However, it was not until 1979 that the first gravitational lens system was actually found.

The first lens system, Q0957+561, was discovered by Walsh et al. (1979) during the course of optical followup of sources found in a radio survey at 966 MHz with the Jodrell Bank telescope. The source object in this system is a radio-loud quasar at redshift 1.41, doubly imaged by a galaxy at redshift 0.36 (Fig. 1). The large (6'') separation of the two images in this system is due to the fact that the lensing galaxy is assisted by a cluster at the same redshift; this allowed Walsh et al. to measure separately the redshifts of the two lensed images, whose spectral similarity supported the hypothesis that their light originates from the same background object. In the next few years, further objects were discovered, many of them in radio surveys; these have the advantage that the sources they contain are predominantly non-thermal emitters such as quasars, without contamination from stellar processes. A typical non-thermal radio source

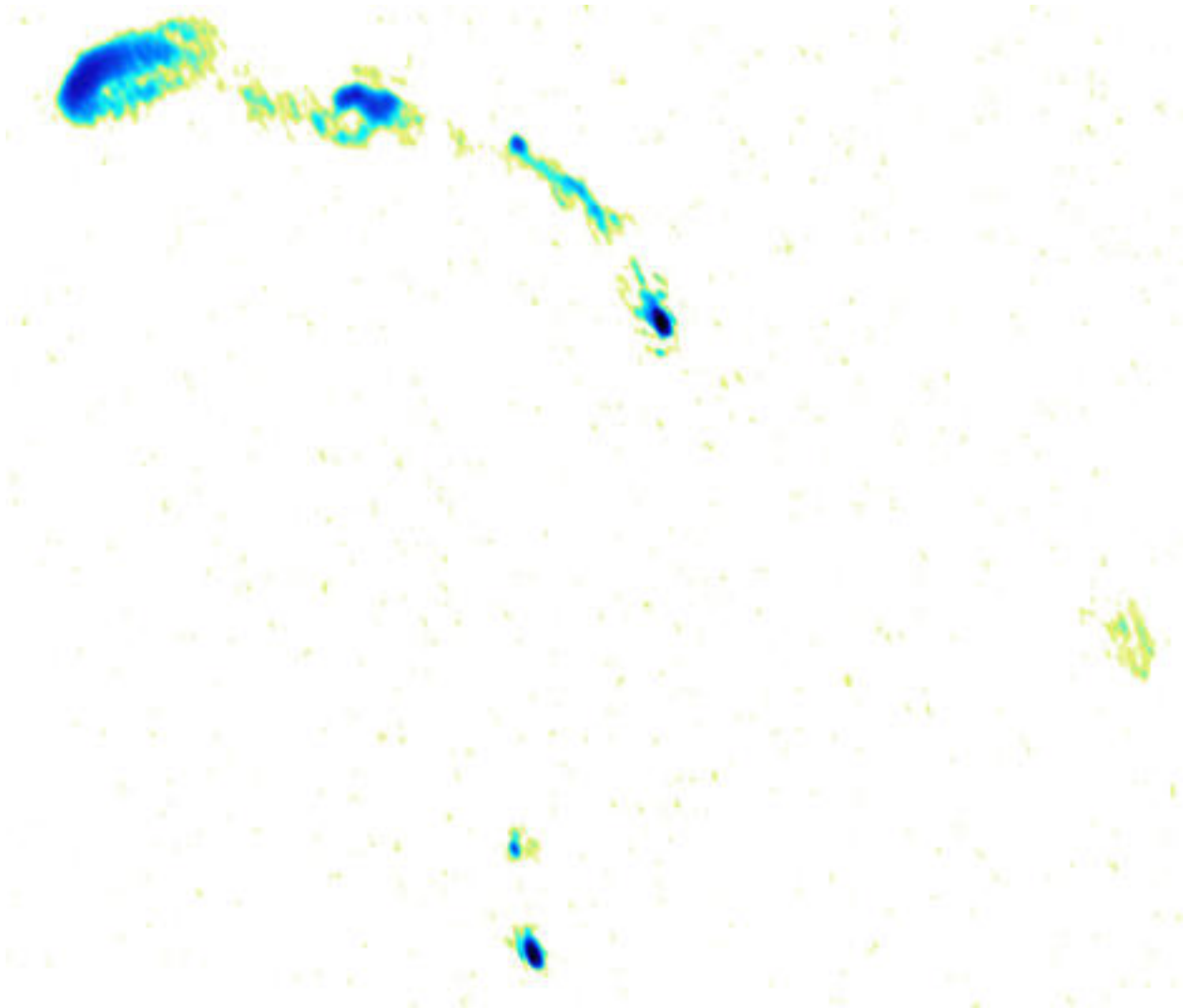


Figure 1: A modern view of the Q0957+561 lens system, made with the e-MERLIN telescope at 5GHz. The quasar consists of a core, radio jet (top left), and lobe (bottom right). The second image of the core (bottom of the image) is close to the lensing galaxy. (Image credit: Muxlow, Beswick & Richards, Manchester).

consists of a central bright flat-spectrum core, corresponding to the active nucleus at the centre of the host galaxy, and extended steep-spectrum emission in lobes which result from ejection of material from the active centre. Although the Q0957+561 system has a double image of the core, further quasar lenses were found in which the extended radio emission was gravitationally imaged, resulting in rings (e.g. MG1131+0456, Hewitt et al. 1988).

Searches for lensed radio sources divided into two main areas. The first, following the success of the MG survey, targeted extended radio lobes, looking for rings associated with the presence of galaxies in front of them (Lehár et al. 2001, Haarsma et al. 2005). The second method involved systematic targeting of flat-spectrum radio sources, in which the central bright point component dominates the radio emission. This makes lensing relatively easy to recognise, although the low optical depth to lensing means that a large number of candidates must be examined to find relatively few lenses. The CLASS survey (Myers et al. 2003, Browne et al. 2003) is still the largest systematic radio survey, and produced 22 lens systems from a parent sample of 16503 northern objects initially observed with the VLA and followed up at higher resolution using

Merlin and VLBI. A southern extension of this survey also exists (e.g. Winn et al. 2001) which discovered a further 4 lens systems. Most of the lenses are elliptical galaxies, since these are generally more massive and hence dominate the lensing cross-section, but a significant minority are later types. Subsamples of these 22 are still important for many of the astrophysical and cosmological applications that will be discussed later in this review.

About 90% of quasars do not have bright radio emission. Searches for lensed radio-quiet quasars began soon after the Walsh et al. (1979) initial discovery, most successfully using the HST. Lensed optical quasars began to be discovered in substantial numbers following the availability of the Sloan Digital Sky Survey (SDSS, York et al. 2000) and subsequently of quasar catalogues derived from it. The Sloan Quasar Lens Search (SQLS; Oguri et al. 2006, Inada et al. 2008, Inada et al. 2010, Inada et al. 2012) has discovered the majority of these, by following up quasars which show extended emission on SDSS images, suggesting the presence of secondary lensed images, or of a lensing galaxy, or both. The SQLS has produced about 30 quasar lenses in the most recent catalogue, including some of the widest-separation lenses known (Inada et al. 2003, Inada et al. 2006). In addition a number of smaller surveys have increased the number of lenses, using various methods. These include the use of higher-image quality supplementary surveys such as UKIDSS (Lawrence et al. 2007) to find small-separation or high-flux-ratio lenses (Jackson et al. 2009, 2012).

The most complete current database, the Masterlens project (Moustakas et al. 2012), lists 120 quasar lenses of which 71 result from two surveys (SQLS and CLASS) and the remainder are serendipitous discoveries or result from smaller surveys. It is this sample that forms the current basis of the scientific results discussed in this review. Future instruments will increase this sample by many orders of magnitude, allowing correspondingly more detailed and wide-ranging science which is discussed in the final section.

QL as probe of lens galaxies

Introduction

Any lens system allows the projected mass distribution of the lens to be probed. Since the lens is typically an elliptical galaxy at significant (~ 0.5) redshift, this is an intrinsically interesting observation, as detailed study of stellar dynamics at this distance is a painful operation involving large telescopes and long integration times. Moreover, lensing gives the projected mass of the lens within the Einstein radius with unique accuracy. There is less good news, however; the probes of the lens potential are obtained only at points where an image is formed, and typically these are limited because the emission from quasars is usually very compact. The resulting problem of mass reconstruction is thus typically underconstrained, giving a serious problem with degeneracies when trying to deduce other mass properties of the lens, particularly the radial mass profile (e.g. Saha 2000, Wucknitz 2002, Kochanek 2002, Kochanek 2004). Probably the most depressing degeneracy is the so-called mass sheet degeneracy (Falco et al. 1985, Gorenstein et al. 1988), which results from the observation that it is possible for an intervening mass distribution to leave the lensed image positions and fluxes undisturbed, while rescaling the overall potentials and time delays together with the (unobservable) source position. This is a potentially serious problem for cosmological investigations.

One approach which alleviates some of the degeneracy problems is to abandon the use of point-source lenses in favour of lensed galaxies, which have extended source structure and thus give constraints at multiple radii in the image plane. This has had considerable success in the investigation both of overall mass profiles of galaxies and of sub-galactic level substructure.

The most significant work in this direction stems from the SLACS survey (Bolton et al. 2006, Koopmans et al. 2006, Bolton et al. 2008). A second approach is to concentrate on a small sample of high-value objects and do the followup observations which are necessary to break the degeneracies; this is mainly to derive additional constraints on the mass profile from observations in regimes where extended sources are visible in addition to the point-like quasar. This approach also requires extensive investigation of the foreground (Fassnacht et al. 2006, Momcheva et al. 2006), in order to derive the information about nearby objects required to ease the mass-sheet degeneracy. (Alternatively, very rare objects, such as lenses with two different sources at different redshifts, can be used to determine the source position and thus break the MSD completely). The third approach is to use quantities derived from the observations which are sensitive to a limited subset of the mass properties of the lens. Both the second and third approaches have been used for quasar lenses.

Lensed substructure

The main attraction of quasar lenses is that they provide a probe of sub-galactic-scale matter structure, which is in turn relevant to a strong prediction of CDM models of structure formation. In such scenarios, structure in the Universe forms in a hierarchical way, with small dark matter clumps coalescing into larger haloes as time passes to form steadily larger agglomerations (White & Rees 1978). Baryons lose potential energy by non-gravitational means and therefore settle in to the resulting potential wells, forming galaxies, groups and clusters. Because many physical processes are involved, how this happens is quite complicated, and in practice semi-empirical recipes are used for describing the process (Blumenthal et al. 1986, Ryden 1988, Gnedin et al. 2004). There are also ongoing processes which may rearrange the baryonic material in galaxy-sized haloes, including the influence of supernovae in lower-mass haloes (Heckman et al. 1990) and of periodic ejections of matter from active nuclei in the centres (e.g. Begelman et al. 1991, Croton et al. 2006); these processes are collectively known as feedback. Complications associated with baryon physics cannot be avoided in lensing systems, because typical gravitationally lensed images form at radii of about an arcsecond, corresponding to 5-10kpc in projection against the lens galaxy. At this radius, dark matter is expected to contribute at the level of a few tens of percent to the projected matter distribution, and baryon processes are therefore dominant.

The substructure debate matters, because CDM works so well on cluster and supercluster scales that its predictions on smaller scales are one of its few possible failure modes. In our own Galaxy, the observation that fewer luminous satellites were found than CDM subhalo models would predict (Moore et al. 1999, Klypin et al. 1999), generated a vast literature reflecting the importance of the problem. Possible ways out of the problem included finding at least some of the missing satellites (Belokurov et al. 2006, 2007; Zucker et al. 2006a,b), or finding some way in which they might be present but not accrete gas and form stars (Bullock et al. 2000). The current situation is that it is difficult to explain both the incidence of substructure and its dynamical properties and stellar content (e.g. Boylan-Kolchin, Bullock & Kaplinghat 2011, Boylan-Kolchin, Bullock & Kaplinghat 2012) although possibilities exist including detailed adjustments corresponding to detailed treatments of the physics, or gross changes such as an alteration in the overall mass of the Milky Way and thus of its expected subhalo content. Curiously, despite the apparent lack of substructure on sub-Magellanic Cloud mass scales, the presence of substructure on scales as massive as the Magellanic Clouds themselves is mildly anomalous, unless the Milky Way has a mass towards the upper end of the allowed range (Boylan-Kolchin et al. 2010).

Relief from detailed arguments about CDM substructure in our own Galaxy can be had by considering substructure probes in other galaxies, in which details can be swept under the observational carpet - or less cynically, a larger number of objects can be probed in less detail, thus accounting for the possibility that our Galaxy may be untypical. The main mass probe in other galaxies at cosmological distance is gravitational lensing.

The observation of a gravitational lens system yields a set of observed positions and flux densities of lensed images. As already discussed, this set of observables does not give unique information about the mass distribution, and in general a large number of macromodels (a term generally used for overall mass distributions, or at any rate for components of spatial frequency in mass distribution of a few kpc or larger) are compatible with images of a single object. Some plausibility arguments can be used to restrict the available set of macromodels, mainly the observation that well-constrained lens systems, with stellar dynamics and extended images, suggest approximately isothermal¹ distributions of mass (e.g. Cohn et al. 2001, Rusin et al. 2003, Koopmans et al. 2006). This density profile corresponds to a flat rotation curve. On top of the macromodel, any smaller-scale perturbations will affect the image positions and fluxes. Since image positions depend on the first derivative of the projected potential distribution and fluxes on the second, smaller perturbations are expected to be detectable in image fluxes.

A number of relations between fluxes of individual lensed images exist which are independent of, or at least relatively insensitive to, the details of the macromodel. For example, “cusp” configuration lens systems, in which the source lies close to the cusp of the astroid caustic produced by the lens galaxy (Fig. 2), produce three bright images close together on the opposite side of the Einstein ring from a single faint image. The brightness of the central image of these three should be equal to the combined brightness of the outer two (Schneider & Weiss 1992; see also Keeton, Gaudi & Petters 2003, 2005; Congdon, Keeton & Nordgren 2008 for more detailed treatment of other cases), and any departure from this indicates a non-smooth mass model. The relation holds because the images form at a very similar, and relatively flat, part of the Fermat surface² in which unphysically large changes in the macromodel would be needed to produce disagreements – known in the literature as “cusp violations” – with the expected relation. On the other hand, small-scale structure can produce cusp violations relatively easily. Constraining small-scale structure is in principle a matter of counting the number and magnitude of cusp violations in a sample of quasar lenses.

There are a number of reasons why the problem is not that easy. The first is that anomalous fluxes can be produced not only by CDM substructure on 10^6 - $10^9 M_\odot$ scales, but also by the movements of individual stars in the lensing galaxy which create a caustic pattern of differential magnification which tracks across the field in timescales of years, with individual events happening on shorter timescales. This phenomenon, microlensing, described in detail later, is itself extraordinarily interesting, but a contamination for the current purpose. It can be got around by using sources which have large sizes relative to the scale of the microlensing caustic

¹This corresponds to a surface density profile $\Sigma \propto r^{-1}$, or a 3-dimensional density profile $\rho \propto r^{-2}$.

²The Fermat surface is a very useful way of thinking about gravitational lens optics. Imagine a source, viewed by the observer in projection on to the lens plane, with contours drawn according to the light travel time of rays originating in the source, bending in the lens plane and reaching the observer. These contours are simply concentric circles centred on the source, with a central stationary point (a minimum) at which Fermat’s principle dictates the formation of an image. If we then introduce a galaxy, which distorts these contours, we eventually reach a point at which further stationary points (a maximum and saddle point) simultaneously form.

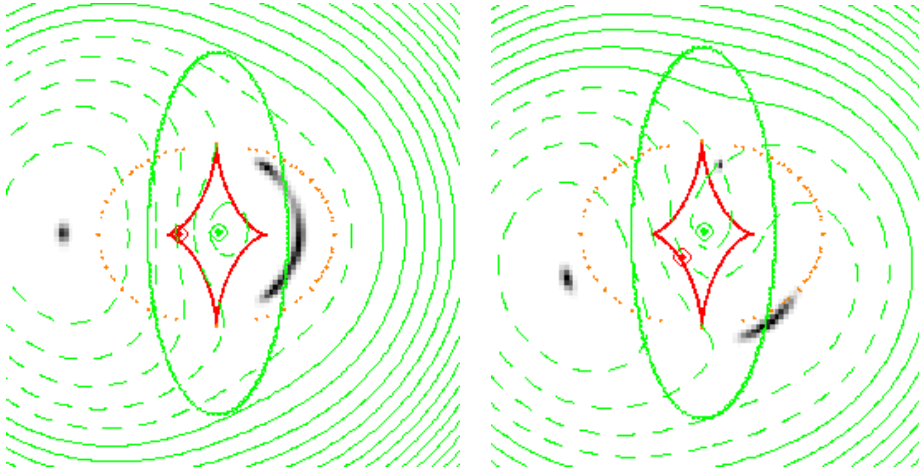


Figure 2: Geometry of cusp (left) and fold (right) lenses. In each case the galaxy is centred on the green dot, and the projected position of the source is the red dot. The astroid caustic (the red diamond-shaped feature) separates 2-image from 4-image systems. The greyscale represents the observed images of the source, which form at positions which are stationary in the time-delay surface (indicated by green contours).

pattern, which in practice is about $1 \mu\text{as}$. The cores and VLBI-scale jets of radio sources, with a typical intrinsic angular size³ of about 1 mas, fulfil this condition, but optical quasars do not. The second problem, even for radio quasars, concerns propagation effects. Radio waves can be scattered while propagating through ionized media; the details are complicated (Rickett 1977) but the effect is to produce flux variations, which can be seen on timescales of weeks to years if the scattering screen is in our own Galaxy. By definition, there is a possible source of foreground screen in gravitational lens systems, in the form of the lensing galaxy, as well as a nearer screen in our own galaxy. Koopmans et al. (2003) found evidence for this in at least one object during a monitoring campaign of some radio lenses, but it is likely to be a small effect compared with most of the observed flux anomalies. The third possible problem is the effects of intrinsic variation of the quasar, coupled with a differential time delay between the images. Even though time delays and flux density variation are useful for measuring the Hubble constant, for the present purpose they are a nuisance. Again, however, the level of variation of most radio sources does not seem to be significant enough to be a major problem, and can be averaged out if observations are made for periods much longer than the time delay. In the optical, extinction is present, and can be used to probe the properties of the dust in the lensing galaxy by using the fact that the same object's light path passes through two different regions of the galaxy (Elíasdóttir et al. 2007).

The first obvious flux anomaly was pointed out by Mao & Schneider (1998) in the CLASS lens system B1422+231; this is a cusp system which produces a violation which requires a significant amount of substructure (about that predicted by CDM) in order to give a significant chance of reproducing the observed anomaly. Other examples of flux anomalies which defied smooth macromodels soon followed (Fassnacht et al. 1999, Metcalf & Zhao 2002, Chiba 2002, Saha, Williams & Ferreras 2007), leading to the first attempt to address the overall statistics (Dalal & Kochanek 2002, see also Kochanek & Dalal 2004). Using seven four-image lenses,

³The size of a compact radio source is controlled by where the optical depth to synchrotron self-absorption becomes 1. This is typically about 1 mas for a source of around 1 Jy, although it becomes smaller with increasing frequency, and it decreases as the square root of the flux. Sources typically found by the Square Kilometre Array, which will be sensitive to sources of $1 \mu\text{Jy}$, may therefore show radio microlensing.

Dalal & Kochanek derived an overall substructure contribution of 0.6-7% (2σ confidence) in substructures between $10^6 M_\odot$ and $10^9 M_\odot$, in rough agreement with the overall predictions of CDM. However, this substructure appears to be in the wrong place (Mao et al. 2004); dark matter, and hence dark matter substructure, should in CDM models be less centrally concentrated than the baryons, and such levels of substructure at projected radii of 5-10 kpc are thus surprisingly high – a curious contrast to the “missing satellite” problem in our own Galaxy. Incidentally, the presence of a tension between lensing observations and CDM probably gives a severe problem for models involving significant amounts of warm dark matter (WDM), which would predict even less substructure (Miranda & Macciò 2007).

A more sophisticated approach to CDM testing can be taken, if instead of calculating an average contribution of substructure “expected” at the projected Einstein radius, we instead take an actual CDM halo simulation and investigate its lensing properties. Early attempts, with lower resolution simulations, produced mixed results (Bradač et al. 2004, Macciò et al. 2006, Amara 2006), but generally confirmed the picture of an excess of flux anomalies compared to the expected incidence in Λ CDM. As better simulations became available, they have been used for these comparisons (Xu et al. 2009, see also Chen, Koushiappas & Zentner 2011 for more detailed treatment of halo-to-halo variations) using, for example, the Aquarius dark-matter simulations. There are a number of limitations with such investigations. The two main problems are that the simulations are being pushed to the limit of their resolution, since they are being asked questions about mass condensations on scales down to $10^6 M_\odot$, comparable to the lowest masses considered in the simulations, and that the effect of baryons in modifying the structure of the subhaloes is not taken into account. Until higher-resolution simulations with extra physics are available, however, this is the best that can be done. Xu et al.’s conclusion was that the cusp violations in existing lenses clearly exceeded the level of violation which would be expected in the dark matter simulations. Two important caveats to this emerged in subsequent work, however. Firstly, detection of substructure along the line of sight means just that, and the substructures which produce the flux anomalies do not have to be within the lensing galaxy (Metcalf 2005a,b, Inoue & Takahashi 2012). Xu et al. (2012) suggested that 20-30% of the substructure could be outside the lensing galaxy, somewhere along the line of sight. If correct, this would potentially alleviate the tension between lensing observations and CDM, although it is probably fair to say that more work, both theoretical and observational, is needed before this can be regarded as well established. Secondly, finite source-size effects may modify the statistics of substructure detection (Dobler & Keeton 2006; Metcalf & Amara 2012).

The sample of seven radio-loud lenses used for substructure studies has remained largely unchanged in the last decade, owing to the current difficulty of finding significant numbers of new radio lenses with existing telescopes. There are a number of alternative approaches. The first involves the use of observational brute force; to target radio-quiet lenses, but observe flux densities in parts of the electromagnetic spectrum where the source has significantly greater size than the microlensing characteristic size of $1\mu\text{as}$. The obvious choice is the mid infra-red, where the source is expected to consist of a more extended thermal component than the accretion disk which radiates in the optical and ultraviolet. Despite the difficulties of observing in this waveband, a number of successful programmes have been carried out (Chiba et al. 2005, Fadely & Keeton 2011; Fadely & Keeton 2012) resulting in the detection of a number of other flux anomalies, and measurement of their likely masses. These range from the $10^{7.3} M_\odot$ and $10^{7.7} M_\odot$ clumps found in MG0414+0534 and HE0435–1223 respectively (MacLeod et al. 2012, Fadely & Keeton 2012) to larger perturbations ($10^9 M_\odot$ in SDSS J1029+2623 and $2 \times 10^8 M_\odot$ in 1938+666; Kratzer et al. 2011, Vegetti et al. 2012). By contrast, the substructure identified in galaxy-galaxy lenses is often larger (e.g. Vegetti et al. 2010). Radio-quiet quasars are also not

radio-silent, and flux densities have been measured for a number of such lens systems (Kratzer et al. 2011, Jackson 2011). Indeed, one would expect that all quasars emit measurable flux density at radio frequencies (White et al. 2007) with current instruments such as the EVLA and e-MERLIN.

An alternative approach is to use the presence of additional observational constraints, such as those provided by radio jets in quasars, to give additional observational constraints, in the case where the jets can be detected in more than one lensed image. This was first attempted by Metcalf (2002) in the case of the lens system CLASS B1152+199 (Myers et al. 1999, Rusin et al. 2002) and a detection was claimed in this case. With further investigations using VLBI, other lenses have been shown to require substructure (MacLeod et al. 2012) and this may be a promising path to more detailed substructure measurements in the future, given sensitive VLBI observations and high resolution (Zackrisson et al. 2012). Currently, one of the most puzzling cases is the four-image lens system CLASS B0128+437 (Phillips et al. 2000), in which the source consists of three radio components separated by a few milliarcseconds and resolvable with VLBI. Attempts to fit the positions of the twelve resulting images fail badly (Biggs et al. 2004). In this object, also, the SIE macromodel which properly fits the four images on arcsecond scales contains an implausibly large amount of external shear, which is inconsistent with the observed number of surrounding galaxies, and also does not fit the extended structure around the images seen by adaptive optics observations (Lagattuta et al. 2010). A fruitful area of future investigation may well be to try and combine the flux and astrometric anomalies in a sample of lenses; although in the case of astrometric anomalies, unlike flux anomalies, the anomaly is always likely to be underestimated since it can be absorbed into the macromodel (Chen et al. 2007). An alternative issue for the future is the proposal that time delay measurements may also be useful for measuring the effects of substructure, which can in extreme cases change the sign of the differential time delay between two images (Keeton & Moustakas 2009).

Having detected mass substructures, we can ask whether they consist purely of dark matter or whether they contain stars. In many cases, flux anomalies can be explained by a mass contribution from a substructure which corresponds to an observed luminous satellite galaxy (Schechter & Moore 1993, Ros et al. 2000, McKean et al. 2007, Macleod, Kochanek & Agol 2009), although in some cases (McKean et al. 2007) the mass model of the satellite is contrived, indicating that further mass structures may be needed. The number of bright subsidiary deflexors may be larger than expected from simulations (Shin & Evans 2008), a problem which may be resolvable if some of them are actually line-of-sight structures, or explicable as a selection effect if brighter condensations are rendered more effective at causing flux anomalies because they have higher central densities.

Black holes and central potentials of lens galaxies

A further important astrophysical application of quasar lens systems - and in particular, of radio quasar lens systems - is the detection and study of “odd” images. All gravitational lens systems have an odd number of images, usually 3 or 5, resulting from the properties of the lens Fermat surface. One of these images is always a Fermat maximum which forms very close to the centre of the lens galaxy. For most realistic mass distributions, this maximum in the surface is very sharp, which implies that the corresponding image is very faint (Wallington & Narayan 1993, Rusin & Ma 2001, Keeton 2003). How faint it is depends on the geometry of the lens system and the degree to which the central potential is singular; if the potential is dominated by a massive black hole, the corresponding image can be hugely demagnified and for all practical purposes invisible. The geometry of the lens system has an effect because this

determines the separation of the central image from the centre of the lensing galaxy. Three-image lens systems, particularly those with high primary-secondary flux ratios, create central images further from the lens centre and which are thus less demagnified. Five-image systems, with four bright images, are expected to nearly always contain an undetectably faint fifth image because the symmetry of the lens configuration places it close to the galaxy centre.

Propagation effects are likely to be particularly acute when trying to detect central images, because the light path passes straight through the lens galaxy centre where the concentration of dust and ionized gas is high. The use of radio lenses, where the galaxy is unlikely to be visible, is required, but relies on the expectation or hope that the central image will not be scattered out of existence. Nevertheless, detection of a faint radio image is not the end of the story, since it may result from low-level radio emission from the core of the lensing galaxy. In principle this can be distinguished by observation at different frequencies and examination of the radio spectrum to see if it differs from the other images.

Observations to detect odd images are very difficult, because they require a combination of high resolution and extreme radio sensitivity. A comprehensive theoretical study (Keeton 2003) of likely lens mass profiles, based upon HST observations of Virgo ellipticals (Faber et al. 1997) showed that central image detections were likely only once flux density levels of 10-100 μ Jy were reached. This level is only now becoming routine thanks to high-bandwidth upgrades to the VLA (now the JVLA) and MERLIN (now e-MERLIN). With older instruments, there is only one secure detection of a central image in a galaxy-mass lensing system, namely PMNJ 1632–0033 (Winn et al. 2002, 2003, 2004), although other systems in which a softer cluster potential is the primary deflector have shown central images (SDSS1004+4112, Inada et al. 2005). In other cases, considerable effort with older instruments has yielded only upper limits (Boyce et al. 2006, Zhang et al. 2007).

The central image in PMNJ 1632–0033 implies two limits; on the mass of the central black hole (which must be less than $2 \times 10^8 M_\odot$) and a lower limit on the central surface mass density. These can alternately be rewritten as joint limits on the index of the central mass power law and black hole mass. In principle, the degeneracy between these two parameters can be broken in the case where the third image can itself be split by the combined lensing effect of the black hole and central stellar cusp into two images (Mao, Witt & Koopmans 2001). Such a detection, although very much more difficult and requiring another factor of 10 in sensitivity, would be very exciting because it would enable the immediate measurement of the black hole mass and central stellar cusp density separately. Even the detection of third images, or significant limits thereon, in a number of radio lenses would give a powerful indication of the evolution of the central regions of elliptical galaxies between $z = 0.5$ and the present day.

QL microlensing: a probe of quasar structure

The combined effect of many stars within the lensing galaxy is to produce a maze of caustics, elongated regions of high magnification with dimensions of microarcseconds which form an intricate pattern across which the source moves. Sources with angular sizes smaller than the characteristic scales of this pattern suffer time-dependent magnification as the pattern moves across them, and consequently the brightness of each lensed image varies as its line of sight crosses the caustic pattern. The details of the resulting effects on the image lightcurves were calculated in the years following the discovery of Q0957+561 (Chang & Refsdal 1979, Paczynski 1986, Kayser et al. 1986, Kayser & Refsdal 1989). It was first observed by Irwin et al. (1989) in the lens system Q2237+0305 (the “Einstein cross”, Huchra et al. 1985) which is a four-image lensed system produced by a low-redshift spiral galaxy with a high central stellar density

around the lensed images; the system is also useful because the time delays are small, much less than the timescale of variations due to microlensing.

The most basic information carried by the microlensing lightcurves is a combination of source size and mass of the microlensing objects (Schmidt & Wambsganss 1998, Wyithe et al. 2000, Yonehara 2001, Kochanek 2004). However, the fact that sources of different sizes respond differently to microlensing by the stars in the lens galaxy offers an opportunity to study sources in great detail (Wambsganss & Paczynski 1991), as well as a way to infer the presence of microlensing by differences in spectra between one image and another (e.g. Wisotzki et al. 2003).

The central region of quasars contain an accretion disk close to the central supermassive black hole, with temperatures of over 10000 K and producing hard UV and X-ray emission. Further from the nucleus are found broad-line regions, showing typical velocity widths of a few thousand km s^{-1} ; reverberation mapping studies of local broad-line AGN have yielded typical size scales of a few light-weeks for these areas. On larger scales still are likely to lie tori of material which reprocess the quasar radiation and re-emit photons in the infrared, together with narrow-line emission regions a few hundred parsecs from the centre.

Interesting results began to emerge about a decade ago as lensed quasars were monitored extensively at optical wavebands. It is expected that the source sizes are different in different optical colours, because the temperature of the accretion disk increases as its radius decreases, and this should show up as a chromatic microlensing signal (Wambsganss & Paczynski 1991) which was duly found in observational programmes (e.g. Wisotzki et al. 1993, Claeskens et al. 2001, Burud et al. 2002). This effect can be used to estimate the accretion disk size and structure (Poindexter, Morgan & Kochanek 2008, Morgan et al. 2008, Poindexter et al. 2010, Hutsemekers et al. 2010, Dai et al. 2010, Blackburne et al. 2011, Muñoz et al. 2011); the picture that emerges in some cases is of a scale-size of a few light days and a temperature-radius profile that is consistent with a standard Shakura-Sunyaev thin disk (Poindexter et al. 2008). But this is by no means a universal result, and in many cases the inferred size is bigger or the temperature profile is different. For example, Blackburne et al. (2011) analyse multiwavelength observations of a sample of lensed quasars and find that the microlensing properties of many of the objects imply accretion disk sizes of up to a factor of 10 larger than standard disks.

Comparison of the spectra of the broad emission lines in different images of quasar lens systems have shown that the BLR is also microlensed (Abajas et al. 2002, Richards et al. 2004, Wayth et al. 2005, Keeton et al. 2006, Abajas et al. 2007, Sluse et al. 2007, Hutsemekers et al. 2010, Sluse et al. 2012) as originally predicted 30 years ago (Nemiroff 1988, Schneider & Wambsganss 1990). Like the continuum microlensing studies, these are very important clues to the structure of the emitting object. Results from this work include the determination of the overall size scale of the BLR, ranging from <9 light days in SDSS J0924+0219 (Keeton et al. 2006) to a few light-months in Q2237+0305 (Wayth et al. 2005), but lack of consensus on the structure of the BLR. This is obtained from the microlensing signature from different velocity components within each line; in some cases there is evidence for an ordered, biconical structure (Abajas et al. 2007) but larger surveys (Sluse et al. 2012) seem to show microlensing signals which are largely independent in red and blue wings, suggesting a non-spherical structure for the BLR.

QL as probe of cosmology

Cosmological parameters

Well before the discovery of the first lensed quasar, Refsdal (1964) pointed out that a lensed

quasar system could be used to measure the Hubble constant. The basic idea is simple. Light travels along two or more different paths from the source to the observer, via deflections at different points in the lens plane. The resulting path difference can be measured if the background source is variable by comparing light curves from the two images and multiplying by the speed of light. If the source and lens redshifts are known, this then gives an absolute measure of distance together with redshift in the system, the combination of which gives the distance scale. Fortunately, the expected time delays from typical lens configurations are on timescales of weeks to months. In principle, this method offers a clean determination of the Hubble constant on cosmological scales in a one-step method. Even better, in principle, measurements in a number of different lens systems at different redshifts could also allow measurement of $H(z)$ and hence other cosmological parameters⁴.

Historically, ecstasy at the cosmological prospects after the 1979 detection of Q0957+561 quickly turned to agony, both because of the long path to the secure determination of a time delay in Q0957+561 itself (Kundic et al. 1997), but also following the appreciation of the extent of the major systematic of this method. The systematic is closely related to the problem of determining the macromodel in a lensed system, and is that the derived Hubble constant is effectively degenerate with the macro-properties of the lens model, in the sense that steeper mass profiles produce lower H_0 for a given time delay⁵. Worse still, the mass-sheet degeneracy causes rescaling of the time delay for the same image positions and fluxes, and thus has an effect on H_0 which is unknown in a single-source system, unless a census of all the mass along the line of sight can be taken.

Again there are a number of responses to the problem. One is to abandon the attempt to measure H_0 or cosmological parameters in general, regard them as a solved problem at the level that lenses will constrain, and regard time delays as a means to break degeneracies in mass models by using them together with a “known” H_0 – for example, from the Hubble Key Project measurements of Cepheid variables. Many workers in the field would regard this as an unnecessarily defeatist approach, given three facts: lens H_0 work requires much less high-cost observing time than alternatives; large numbers of time delays will be available in future; and although the lens modelling systematic is a serious systematic, it is only one systematic and not many.

The second response is to investigate a statistical approach. Can the systematic error be reduced to a random error, albeit a large one in an individual object, which can then be beaten down by root- n statistics? This approach again begins with the average properties of the SLACS galaxy-galaxy lenses, which appear to have a mass slope very close to isothermal (Koopmans et al. 2006) and mostly lie at low redshift (<0.3). More recently, a higher-redshift lens sample, known as BELLS (Brownstein et al. 2012, Bolton et al. 2012), has become available from a parent population consisting of the BOSS spectroscopic survey. These show that the mass slope changes slightly with redshift, becoming steeper by a few tenths in the overall power law. Matter along the line of sight is harder to control, but here again a statistical argument may be

⁴Early in the history of lensing, the number of lenses in a complete sample appeared to be a useful way of constraining Λ , because a higher Λ increases lengths at high redshift and hence increases the optical depth to lensing (Kochanek 1996); indeed this was the justification for the thoroughness of the CLASS survey in attempting to get a complete sample of lenses. Although this line of research yielded a fairly clear result of non-zero Λ (Kochanek 1996, Chae et al. 2002, Mitchell et al. 2005, but see also Keeton 2002), the rate at which constraints on dark energy improve is a very slow function of increasing size of lens sample, and it has since been abandoned.

⁵Strictly, the dependence is not directly on the mass profile; it is related to the surface mass density in the annulus between the lensed images (Kochanek 2002, but see also Read, Saha & Macciò 2007).

appropriate; multiple sight-lines through large cosmological simulations can give an indication of the possible range of amount of intervening matter along a particular direction (although not a random direction, since lensing is a process which takes preferentially along more crowded lines of sight). A large Bayesian engine can then be employed to marginalise over nuisance parameters and yield the desired information. Statistical approaches have been taken by a number of authors, including for example Dobke et al. (2009) who calculated the number of time-delay lenses which would be required to constrain cosmological parameters other than H_0 in this way. Similar attempts have been made to investigate H_0 using existing time delay information and best-attempt mass models (Oguri 2007), yielding results around $70 \text{ kms}^{-1}\text{Mpc}^{-1}$ although leaving the uncomfortable feeling that mutually formally incompatible results for different lenses are being shoehorned into a harmonious conclusion. Another approach is to explore the variety of possible lens models using non-parametric methods in order to thereby explore the possible range of H_0 for each system (Saha et al. 2006), again yielding results of $72 \text{ kms}^{-1}\text{Mpc}^{-1}$, with errors of about 15%.

The third, and in my view fruitful response in the long run, is to grit one's teeth and do the hard work in individual cases, in the knowledge that it will get easier as telescopes become more powerful. Two recent examples of the work required are provided by the detailed investigations of the time-delay lenses RXJ1131–1231 and CLASS B1608+656 by Suyu et al. (2010) and Suyu et al. (2012). A time-delay is only the start of such programmes. Other ingredients include deep multi-colour HST imaging, in order to properly model the extended structure associated with lensed extended emission from the quasar host and disentangle it from effects of reddening in the lensing galaxy; spectroscopic investigation of surrounding matter, in order to quantify the effects of mass sheets, taking into consideration the richness of the surrounding environment and comparison with cosmological simulations; radio imaging, in the case where the lensed object is radio-loud; and performing the modelling blind to avoid unconscious biases, so that the conversion from an arbitrary scale to H_0 is only done once all the systematics have been estimated. However, when all this is done, the resulting H_0 values from two lenses, including all the systematic and random errors, are of comparable quality to the HST Key Programme (6% error). There is a serious prospect from further such studies of a competitive contribution to the determination of w which is largely orthogonal to other probes, once a few dozen such lenses have been thoroughly investigated (e.g. Linder 2011).

Galaxy evolution

Individual lenses can be used to find values for the Hubble constant (or for ambiguity-free galaxy mass models if the cosmological world model is known). However, the statistics of well-selected lens samples can be used to investigate galaxy evolution. This is because the number of lenses as a function of galaxy and source redshift in such a sample depends on the evolution in both density and mass of the available lens population. In order to attempt this, a sample whose selection effects are under control is needed, and in practice the easily identifiable properties of quasars, and the possibility of getting clean separation between lensed and unlensed objects, makes quasar lens samples the obvious choice.

Two at least fairly complete quasar lens samples exist. The CLASS statistically complete survey was the result of a systematic attempt to completely identify all lenses with separation of $>300 \text{ mas}$ and primary:secondary flux ratio $<10:1$. The SQLS survey, although probably somewhat less complete at the lower-separation end, is slightly larger. A number of authors, beginning with Chae & Mao 2003, have used these samples, together with a plausible cosmological world model, to derive useful constraints on early-type galaxy evolution. Most have

found a consistency with no evolution either in number density or mass (Chae & Mao 2003, Chae 2005, Chae, Mao & Kang 2006, Matsumoto & Futamase 2008, Chae 2010, Oguri et al. 2012), although due to the small statistics, the error bars are still large in the index of number density evolution. In the most recent work (Oguri et al. 2012), the evolution index of velocity dispersion $\nu_\sigma \equiv d\ln\sigma/d\ln(1+z)$, is zero within errors of ~ 0.2 , assuming a standard Λ -cosmology.

The future; bigger samples, and how to find them

The current largest sample of quasars, the SDSS quasar list (Schneider et al. 2010) contains about 100000 objects. Given a typical optical depth in lensing galaxies towards $z \sim 2$ sources, this would imply 100-150 lensed quasars, of which at least half have already been found in the SQLS and other surveys. In principle, many more active galaxies are available in radio surveys - the FIRST survey, for example, contains about 10^6 objects with accurate position information - but their faintness in the radio makes systematic large surveys difficult, even with the current generation of upgraded radio arrays. Of such arrays, the only one which has the required combination of high sensitivity and sub-arcsecond resolution is LOFAR, a low-frequency radio interferometer array centred in the Netherlands (van Haarlem 2005) but high resolution surveys at great depth are probably a few years away.

In the optical, a combination of wide area and high resolution is likely to be achieved in the near future, by a number of telescopes. The first is GAIA, scheduled for 2013, a satellite primarily designed for astrometry and measurements of proper motions in Galactic stars. By virtue of area coverage and accurate measurements of point sources, however, it is also well suited to making a large census of about half a million quasars, to determine which such objects are extended and thus potentially lensed. This alone should increase the lensed quasar sample by a large factor (Surdej et al. 2002). Towards the end of the decade, two major advances are likely with the advent of the Large Synoptic Survey Telescope (LSST) and Euclid. Euclid is an ESA medium mission scheduled for launch in 2019 which will have close to all-sky coverage and 150-200 mas resolution at optical and NIR wavebands. It will provide imaging at slightly less angular resolution and sensitivity than the 2 square-degree COSMOS HST field, but over the whole sky. Its mission includes the detection of about 1000 quasar lenses, about an order of magnitude increase on the present sample (as well as hundreds of thousands of galaxy-galaxy lens systems). Still vaster samples will be provided by LSST (Abell et al. 2009), with the additional advantage of multiple observations of the same field, allowing quasars to be identified by variability (Kochanek et al. 2006) and probably yielding a sample of several thousand lensed quasars. On the same timescale, the Square Kilometre Array will provide similarly large samples, selected at radio wavelengths (Koopmans et al. 2004).

Large samples are good for two reasons. The first is that “more of the same” approaches can be tried with larger numbers, although they do rely on systematic biases being eliminated. If we assume that the mass slope and mass sheet model degeneracies can be controlled, so that the accuracy improves as the square root of the number of objects, then spectacular results can be obtained; Coe & Moustakas (2009) calculate that, in conjunction with Planck priors, an accuracy of $\sim 3\%$ can be obtained on w . This assumes that time delays will be measurable given the LSST cadence. If a smaller sample of the quasar lenses are measured, however, but with more intensive followup, then extrapolation from the work of Suyu et al. (2012) suggests that comparable results on post- H_0 parameters can be achieved in conjunction with other (BAO/SNe) cosmological probes, since the lensing constraints are often orthogonal to others in parameter space. Linder (2011) calculates that the dark energy figure of merit is potentially

improvable by a factor of 5 by including lensing information from future surveys.

The second advantage of large samples is that they are likely to contain a small number of high-value objects. One of the most prized type of lens systems is a quasar lens system with a second source at a different redshift, since such double source plane systems allow mass model degeneracies to be immediately broken. The problem has been investigated by Collett et al. 2012, who find that a small number of these rare objects give 15% accuracy in w very quickly. Many such sources would be currently difficult or impossible to follow up, due to faintness of some of the sources, but the era of 30-m class telescopes is around the corner, and such followup operations will become routine if not easy.

Summary and conclusion

The future uses of quasar lenses, as in the past, divide into three natural applications: study of the lenses, study of the sources, and study of cosmology and galaxy evolution. I briefly summarise the results from the main body of the review, and the prospects for the next ten years.

- We already know basic facts about lens mass distributions; elliptical galaxies have isothermal mass distributions at low redshift, and there is some indication of steepening with redshift. In prospect is a vast increase in parameter space, with the ability to study the evolution of galaxy mass profiles over a much wider range of redshift, and over different masses of lens galaxies and Hubble types. Much of this work will be done with lens-selected surveys, like the existing SLACS galaxy-galaxy lenses; however, quasar lens systems give the opportunity to make large, source-selected surveys and examine the statistical properties of lenses independently of their selection. The major impact will be in the study of substructure in lens galaxies, however. The existing sample of substructure-friendly quasar lenses is very small and has already yielded a large and surprising body of information about the small-scale features of lensing galaxies. Expansion of these samples will provide critical tests for galaxy formation models.
- Microlensing studies have yielded unprecedented insights into the nature of quasars, particularly the proportions of stellar and smooth matter within the mass budget and the properties – size and physics – of the central engine and surrounding emission line regions. These studies require multi-wavelength observations and patient monitoring, and have concentrated on relatively few objects. With future telescopes and high-cadence monitoring we can expect that studies of quasar physics at huge effective resolution will become routine.
- Perhaps the most important future application of quasar lensing lies in a promise which has taken some time to fulfil, that of cosmography. Many years passed between the discovery of the first quasar lens and the first reliable determinations of the Hubble constant. The process is now accelerating, thanks to coordinated and long-term monitoring campaigns, allied to advances in lens modelling and observation of individual lens systems. Such investigations are already closing in on estimates of the Hubble constant with stringent enough error constraints to contribute to the overall cosmological world model. Future measurements on a large sample of quasar lenses will give cosmological parameter estimates with error circles orthogonal to many others, and factors of several increase in figures of merit for dark energy searches.
- Finally, the prospect of huge lens samples carries with it the probability of finding some-

thing totally new. We can speculate about the likelihood of possible candidates - completely dark lenses, cosmic strings - but with the expectation that the unexpected is likely to prove more surprising still.

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